Coulomb's Law

Recall from **Coulomb's Law of Force** that a charge Q_2 located at point $\bar{r}_2$ applies a force $\mathbf{F}_1$ on charge Q_1 (located at point $\bar{r}_1$):

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\,\hat{a}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\;\frac{\bar{r}_1 - \bar{r}_2}{\left|\bar{r}_1 - \bar{r}_2\right|^3}$$

Likewise, from the **Lorentz Force Law**, we know that the force $\mathbf{F}_1$ on a charge Q_1 located at point $\bar{r}_1$ is attributed to an **electric field** located at $\bar{r}_1$:

$$\mathbf{F}_1 = Q_1\,\mathbf{E}(\bar{r}_1) \quad\Rightarrow\quad \mathbf{E}(\bar{r}_1) = \frac{\mathbf{F}_1}{Q_1}$$

Inserting Coulomb's Law of Force into this equation, we get the electric field at location $\bar{r}_1$, generated by charge Q_2 located at $\bar{r}_2$!

$$\mathbf{E}(\bar{r}) = \frac{\mathbf{F}_1}{Q_1} = \frac{Q_2}{4\pi\varepsilon_0}\frac{\hat{a}_{21}}{R^2}$$

In general, we can say the electric field $\mathbf{E}(\bar{r})$ at location $\bar{r}$, generated by a charge Q at point $\bar{r}'$, is:

$$\boxed{\mathbf{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0}\,\frac{\hat{a}_R}{R^2} = \frac{Q}{4\pi\varepsilon_0}\,\frac{\bar{r}-\bar{r}'}{|\bar{r}-\bar{r}'|^3}}$$

This is **Coulomb's Law** !! It describes the electric field $\mathbf{E}(\bar{r})$ at location $\bar{r}$ that is created by a charge Q at location $\bar{r}'$.

Note that:

$$\hat{a}_R \doteq \frac{\bar{r} - \bar{r}'}{|\bar{r} - \bar{r}'|}$$

Therefore, **if** the charge Q is at the **origin** (i.e., $\bar{r}' = 0$), then:

$$\hat{a}_R = \frac{\bar{r}}{|\bar{r}|} = \hat{a}_r$$

Recall that the base vector $\hat{a}_r$ always **points away** from the origin. In other words, a charge located at the origin creates an electric field vector that points in the direction of base vector $\hat{a}_r$ (i.e., **away from the origin**) at all points $\bar{r}$!

Likewise, **if** the charge is at the origin, then:

$$R = |\bar{r}| = r$$

In other words, the **magnitude** of the electric field vector is **proportional** to $1/r^2$. As a result, the magnitude of the electric field is dependent on its distance from the origin (i.e., distance from the charge). Therefore, **if** $\bar{r}'=0$**:**

$$\mathbf{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{a}_r}{r^2} = \frac{Q}{4\pi\varepsilon_0}\frac{\bar{r}}{r^3}$$

Q: *What is the **curl** of* $\mathbf{E}(\bar{r})$ *??*

A: $\nabla \times \mathbf{E}(\bar{r}) =$